On Optimal Taxation in the Presence of *Rule-of-Thumb* Consumers

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Abstract

In this paper, I examine optimal taxation in a Neoclassical framework in which a fraction of households do not save or own capital, otherwise referred to as *Rule-of-Thumb* (ROT) consumers. I show that, first, for economies with larger shares of the ROT consumers, the optimal tax structure is weighted less towards income taxes (and more towards consumption taxes) compared to economies with smaller shares of these households. Secondly, the presence of the ROT consumers constitutes a distortion in the capital market, which calls for an increase in the capital stock. To engender a rise in capital formation, the capital income tax is set below zero (a subsidy). Moreover, volatility in the labor income tax is contingent on whether consumption is taxed or not. Introducing consumption taxes gives rise to further volatilities in the tax wedge. To smooth out this wedge, it becomes necessary to adjust the labor income tax over the business cycle.

JEL codes: E12, E62, F41, H63

Keywords: optimal taxation; tax smoothing; capital ownership.

1 Introduction

Since the fundamental work of Ramsey (1927) and Mirrlees (1971), there have been a lot of work on the optimal design of taxes over the business cycle. Some of the notable contributions in this regard are Phelps (1973), Sadka (1976), Judd (1985), Chamley (1986) and Saez (2001), among others. For a comprehensive review of optimal tax theory over the last few decades see Mankiw *et al* (2009). Although, these studies diverge considerably

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in terms of scope, they primarily focus on the size and smoothing properties of taxes. One result that features prominently in the literature is that capital income should be taxed close to zero in the long run, (see for example, Judd, 1985; Chamley, 1986; and Atkeson *et al*, 1999). The reasoning behind this is that, not only does capital income tax depress savings, it also constitutes an inter temporal wedge between current and future consumption. A zero capital income tax therefore eliminates this wedge and ensures efficiency. Several other papers, under different contexts, find the zero capital income tax condition sub-optimal. Such papers include Jones et al. (1993), Jones et al. (1997), Aiyagari (1995), Correia (1996), Judd (2002) and recently, Cerda and Saravia (2013) and Abo-Zaid (2014).

With regards to the labor income tax, much of the literature argue in favor of smoothing the tax rate over the business cycle. For example, Chari et. al (1991), while supporting the idea of a close to zero capital income tax, also argue in favor of smoothing the labor income tax rate over the business cycle. The smoothing properties of the labor income tax, among other things, is consistent with the concept of wedge smoothing, which emphasizes smoothing the tax wedge over the business cycle.

Admittedly, literature on optimal taxation is nearly exhaustive. However, there are still some issues that lend themselves to further scrutiny. One of such compelling issues is the dynamic properties of the optimal tax structure in the context of an evolving economy. Changes in the characteristics of the economy might elicit changes in the optimal tax structure. For example, a tax system that is optimal for an economy with a high savings rate might be sub-optimal otherwise. Similarly, changes in the composition of households in terms of ownership of capital might engender changes in the optimal tax structure. Moreover, a tax policy that is weighted more towards income taxes might generate different macroeconomic effects, especially with regards to savings and investment, than a policy that is weighted more towards consumption taxes.

In this paper, I explore the dynamic properties of the long run optimal tax structure in a neoclassical framework in which households are segregated by capital ownership. Specifically, I examine changes in the optimal tax structure in response to changes in the composition of households in terms of the fraction that save and own capital (*savers*) relative to the fraction that do not, otherwise refereed to as *Rule-of-Thumb (ROT)* consumers. In doing so, I compare the size and smoothing properties of consumption tax, the labor income tax and capital income tax across economies with varying shares of the ROT consumers. The presence of these households has implications for the transmission of tax policy in the macroeconomy, owing to the resultant impact on the average propensity to save in the economy. The findings are summarized in what follows.

First, for economies with larger shares of the ROT consumers, the optimal tax structure is weighted less towards income taxes (and more towards consumption taxes) relative to economies with smaller shares of these households. That is, both labor income and capital income are taxed at a lower rate relative to the economy with a smaller share of the ROT consumers. Consumption, on the other hand, is taxed at a higher rate. This is a central outcome in this paper. The average savings rate is lower if the share of the ROT consumers is larger, *ceteris paribus*. To induce more savings by the relatively fewer *saver* households (and lower the potential crowding out of investment), tax policy is designed to put less emphasis on income taxes as opposed to consumption taxes. Consumption taxes are generally less inimical to capital formation than income taxes.

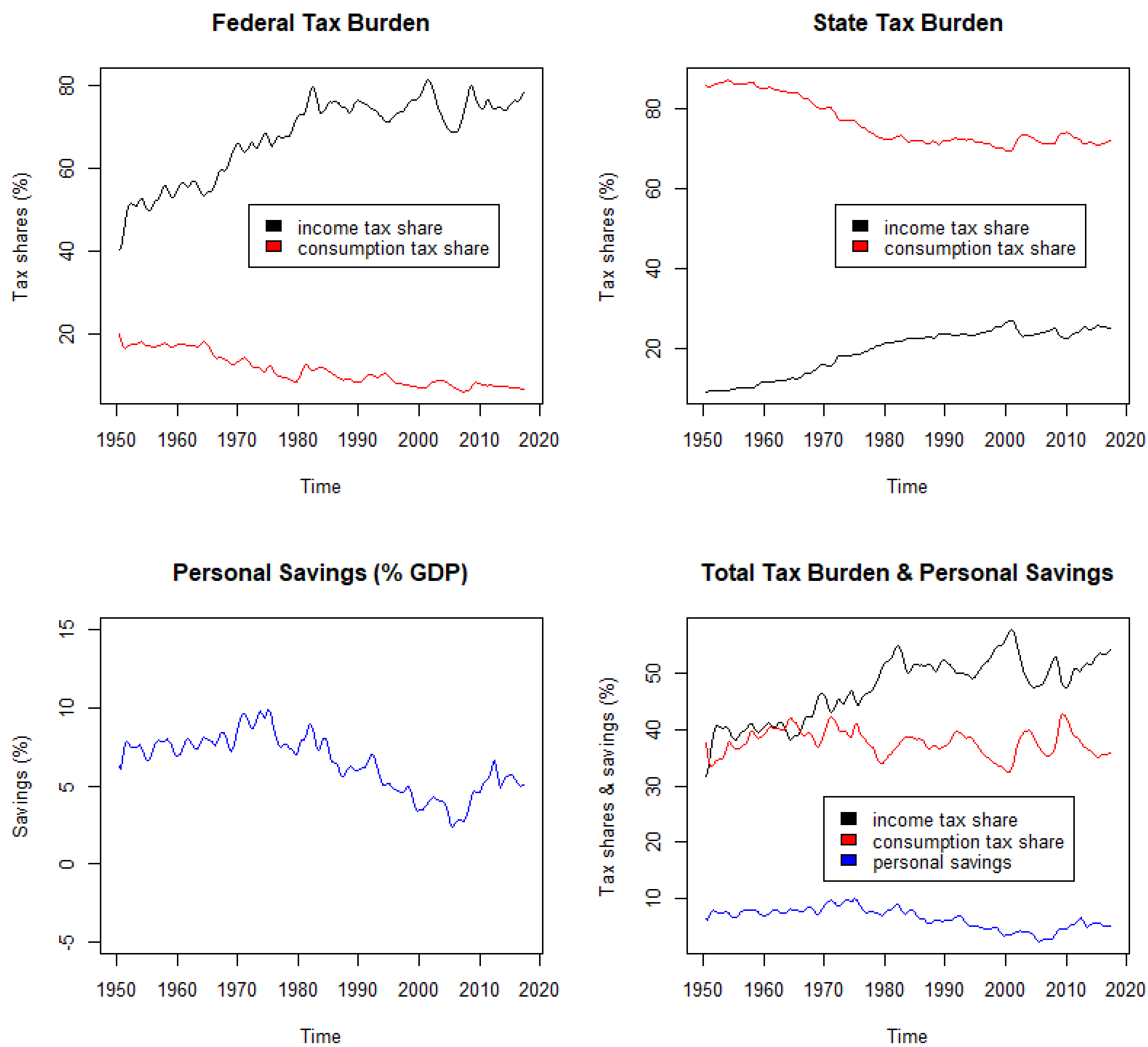
Secondly, smoothing properties of the labor income tax is contingent on whether consumption is taxed or not. In the standard model without consumption tax, the optimal labor income tax rate is constant over the business cycle. In this economy, the labor income tax constitutes the only source of the wedge between the Marginal Rate of Substitution (MRS) and the Marginal Product of Labor (MPN). In order to smooth out this wedge over the business cycles, it is essential that the labor income tax is kept constant. However, once we introduce a tax on consumption, the constant labor income tax condition becomes sub-optimal. And, this is true irrespective of whether ROT consumers are present or not. Changes in the labor income tax over the business cycle offsets the excess volatility in the tax wedge which is imposed by the presence of the consumption tax.

Also, in the standard model without ROT consumers, the optimal capital income tax rate is zero in the long run, which is consistent with the findings in Judd (1985), Chamley (1986) and Atkeson *et al* (1999) among others. However, once we account for the ROT consumers, the optimal capital income tax rate is not only negative, it also falls for larger shares of these households. The presence of the ROT consumers constitutes a wedge between the real rental rate of capital and the marginal product of capital (MPK), which calls for an increase in the capital stock in the economy. To achieve this, the capital income tax is set below zero in order to encourage investment, which ultimately leads to more capital.

1.1 Tax Policy, the Savings Rate and Capital Formation

Capital, in a conventional sense, is a combination of assets that are invested or available for investment. These include stocks, bonds, equipment, among other things. Like many countries, a significant number of households in the United States (US) do not participate in the capital market. Depending on how we measure capital, this number could vary considerably. According to the *Survey of Consumer Finances (SCF)* data published by the Federal Reserve Board (2017), about 86% of households do not hold stocks, 99% do not hold bonds, whereas 87% do not have any form of business equity. On a more conservative side, about 10% of households had zero net-worth or less, whereas about 1.5% did not have any form of financial assets, which includes basic transaction accounts such as checking accounts.

Figure 1: Time Plots of Federal and State Tax Burden and Personal Savings for the US Economy (1950-2017)



Notes: These are trends from quarterly US data spanning 1950:1 to 2017:4. The data is sourced from the US Bureau of Economic Analysis (BEA). Income taxes include personal current taxes. Consumption taxes include taxes on production and imports which, for the federal government, include excise taxes and custom duties. For the States, these include sales tax, excise tax and property taxes. Total tax burden is \the sum of state and federal tax revenue. Taxes are expressed as a percentage of total revenue (burden) for the respective category, whereas personal savings is expressed as a percentage of GDP.

There are varying reasons for which individuals do not hold any form of financial assets.

As noted by Gali, et al (2007), one may attribute this behavior to a combination of factors including myopia, inaccessibility to financial markets and borrowing constraints. According to a 2016 estimates by the US Census Bureau (2016), about 12.7% of people in the United States lived in poverty. Most of these people are credit constrained and do not save. The ROT consumers in this paper are similar to these credit constrained individuals who do not invest in any form of capital. As indicated earlier, the share of these households in the economy has implications for aggregate savings and, ultimately, capital per worker in the long run, which the design of tax policy has to take into account.

Before proceeding to the theoretical framework, I look at the empirical relationships between tax policy, the savings rate and capital investment. Figure 1 presents time plots of income taxes, consumption taxes and personal savings for the US economy. Taxes are expressed as a percentage of total tax burden in the respective category. Personal savings, on the other hand, is expressed as a percentage of GDP. The data shows that, historically, US tax policy at the state and federal level has shifted in favor of more income taxes relative to consumption taxes. That is, the share of tax burden resulting from income taxes has been rising, whereas that from consumption taxes has been falling, irrespective of the level of government (see top two panels in Figure 1). For personal savings, it has been relatively flat from the 1950s through the early part of the 1970s, after which the trend has been downward until the middle of the 2000s.

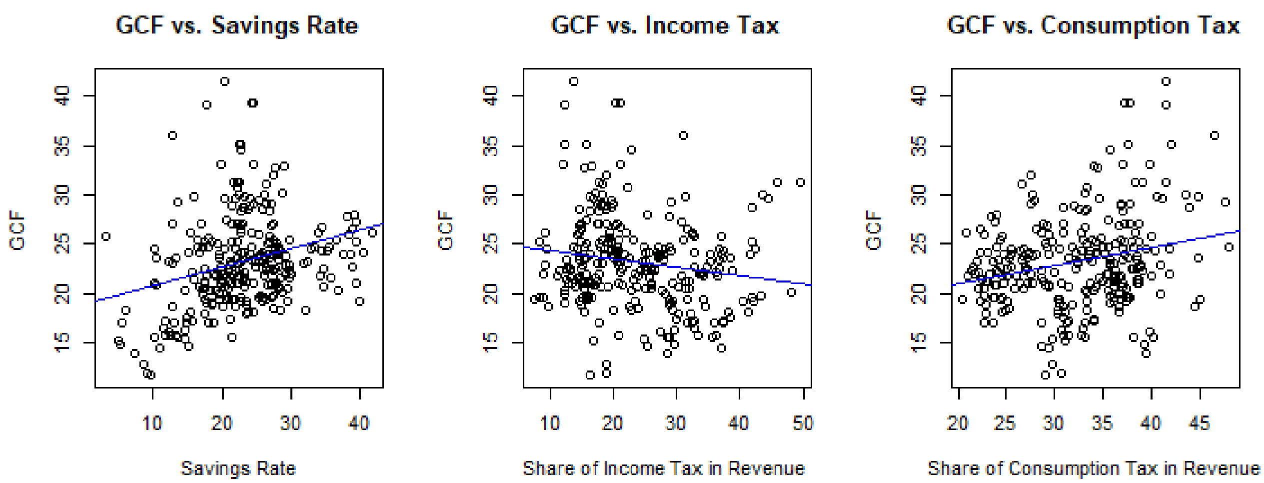
Table 1: Moment Conditions of Income and Consumption Tax Shares in the US (1950-2017)

	Federal	State	Total
Share of income tax in tax burden			
Mean (%)	67.42	19.21	47.46
Standard deviation	9.94	5.70	5.74
Corr with personal savings	−.46	−.60	−.50
Share of consumption tax in tax burden			
Mean (%)	11.19	76.43	37.75
Standard deviation	3.98	5.93	2.39
Corr with personal savings	.63	.55	.42

Notes: Income taxes includes personal current taxes. At the federal level, consumption taxes include excise taxes and custom duties. At the state level these include sales tax, excise tax and property tax. Total taxes include the sum of state and federal tax revenue. Taxes are expressed as a percentage of total revenue for the respective category. Personal savings is expressed as a percentage of GDP. These is constructed from quarterly US data, 1950:1 to 2017:4. The data is sourced from the US Bureau of Economic Analysis (BEA).

Moreover, considered separately, the relationship between state and federal tax burden and personal savings is not clear-cut, especially from 1950 through 1970. In order to get a better sense of the relationship, I consider the total tax burden on tax payers, which is a combination of state and federal tax burden. This is presented in the bottom right panel of Figure 1. From this panel, it is shown that, from the 1950s through the early part of 1970s, the share of total tax burden from income and consumption had been intertwined (and not far apart). Interestingly, trends in personal savings had been quite flat for this same period. However, as the share of income taxes in the total tax burden rises relative to that of consumption taxes, personal savings starts to trend downward. Although we cannot infer causality between personal savings and the share of tax burden from both income taxes and consumption taxes, these relationships hint of a degree of correlation between these variables. As shown in Table 1, the correlation between income tax shares and personal savings is negative, whereas that of consumption taxes and personal savings is positive, regardless of the level of government.

Figure 2: Gross Capital Formation (GCF), the Saving Rate and Tax Burden for a Selected OECD Countries.



Notes: The variable "Income tax" is the sum of taxes on income, profits and capital gains expressed as a percentage of total tax revenue. "Consumption tax" on the other hand is the sum of taxes on goods and services, also expressed as a percentage of total tax revenue. The savings rate is measured as the share of gross domestic savings to total income, whereas GCF is gross domestic investment expressed as a percentage of GDP. These are vector plots constructed from annual data of 25 OECD countries from 2004 and 2014. The data is obtained from the World Bank Country Statistics database.

Next, I present a cross-country evidence of correlation between income taxes, consumption taxes and the savings rate. Figure 2 presents the relationship between shares of income and consumption taxes in total tax burden, the savings rate and gross capital formation (GCF) for a selected OECD countries. The results show that countries with higher savings rates tend to be associated with higher gross capital formation (see the left panel). While this observation is trivial, it points to the potential differences in how capital in these economies might respond to alternative combinations of tax instruments (through differences in the savings rate). Moreover, as shown in the middle and right panels, countries that implement higher income taxes relative to total tax revenues tend to have lower gross capital formation, whereas countries that implement higher consumption taxes relative to total tax revenue tend to generate the opposite effect. This can be loosely considered as an indication of the depressing effects of income taxes on capital formation as opposed to consumption taxes.

The rest of the paper is organized as follows: Section 2 introduces the model; section 3 presents the Ramsey planner's problem; section 4 discusses the analytical results of the paper; section 5 presents the quantitative results whereas section 6 concludes the paper.

2 The Model

The model is composed of a continuum of infinitely lived households who derive utility from consumption and leisure. Households in this economy are categorized into two based on ownership of capital. On the one hand, there are Ricardian households (*savers*) of mass $(1-\omega)$, who save and ultimately own capital. The other category of households, referred to as *Rule-of-Thumb* (*ROT*) consumers, do not save, borrow or own any form of capital. These households are of mass ω. Moreover, there is a production sector which is composed of firms that are perfectly competitive. These firms hire labor and capital from the household sector and produce goods using a specific Cobb-Douglass technology. Also, there is a government which generates revenue from a mix of taxes, proceeds of which are used for government expenditure. Finally, there is a central planner who, given household preferences and competitive market outcomes, maximizes welfare in the economy.

2.1 The Household Sector

Households in this economy are categorized into two based on ownership of capital. Households of category S save and participate in the capital market. Given the amount of labor and capital available to these households, they face the following budget constraint.

$$(1+\tau_t^c)\,C_{S,t}+K_{t+1}\leq(1-\tau_t^n)\,w_{St}N_{S,t}+\left\{(1-\delta)+r_t\left(1-\tau_t^k\right)\right\}K_t \tag{1}$$

where $C_{S,t}$ is consumption, $N_{S,t}$ is labor, whereas K_t is how much capital each household in this category holds. Also, τ_t^c, τ_t^n and τ_t^k are taxes on consumption, labor income, and capital income respectively. $w_{S,t}$ is the real wage for this category of household, r_t is the return rate on capital, whereas δ is the capital depreciation rate. It follows therefore that

$\left\{(1-\delta)+r_t\left(1-\tau_t^k\right)\right\} K_t$ is the real after tax income from capital investment, whereas $(1-\tau_t^n)\, w_{St} N_{S,t}$ is the real after tax income from labor supply. Moreover, given investment per household I_t, capital accumulation by each household in this category is determined as

$$K_{t+1} = (1-\delta)\, K_t + I_t \tag{2}$$

The other category of households (category H) are the so-called *Rule-of-Thumb* consumers who are Non-Ricardian; they do not save or own own capital. The corresponding budget constraint for this type of households is given as

$$(1+\tau_t^c)\, C_{H,t} \leq (1-\tau_t^n)\, w_{Ht} N_{H,t} \tag{3}$$

$C_{H,t}$ is consumption for these households, $N_{H,t}$ is labor supply, whereas $w_{H,t}$ is the real wage. The term $(1-\tau_t^n)\, w_{H,t} N_{H,t}$ is therefore the total after tax real labor income for this household category. In addition, household preferences are determined by the following.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t U_{i,t}\left(C_{i,t}, N_{i,t}\right) \tag{4}$$

where subscript $i = S, H$ represents *savers* and ROT consumers respectively. In each period, a representative household of category i, given the inter-temporal discount rate β_i^t, chooses consumption $C_{i,t}$, labor $N_{i,t}$, capital K_t (where available) to maximize the following expected lifetime utility subject to the respective budget constraint represented by equations (1) and (3). Solving the household's problem, and assuming symmetry within each household category yields the following optimality conditions.

$$\lambda_{S,t} = \frac{U_{CS,t}}{(1+\tau_t^c)} \tag{5}$$

$$\lambda_{H,t} = \frac{U_{CH,t}}{(1+\tau_t^c)} \tag{6}$$

$$\frac{-U_{NS,t}}{U_{CS,t}} = \left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) w_{St} \tag{7}$$

$$\frac{-U_{NH,t}}{U_{CH,t}} = \left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) w_{Ht} \tag{8}$$

$$1 = \beta_S \mathbb{E}_t \left(\frac{\lambda_{St+1}}{\lambda_{St}}\right) \left[(1-\delta) + r_{t+1}\left(1-\tau_{t+1}^k\right)\right] \tag{9}$$

$$C_{H,t} = \left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) w_t N_{H,t} \tag{10}$$

where $\lambda_{S,t}$ is the Lagrange multiplier on equation (1) and $\lambda_{H,t}$, the Lagrange multiplier on equation (3). $U_{CS,t}$ and $U_{CH,t}$ are the marginal utility of consumption for *savers* and ROT consumers respectively. $U_{NS,t}$ and $U_{NH,t}$, on the other hand, are the marginal utility of labor for these households respectively.

2.2 Firms

Firms in this sector are perfectly competitive. Each firm j employs workers from both types of households and rents capital from the *saver* households to produce good $Y_t(j)$, which is determined by the following production technology.

$$Y_t(j) = Z_t F\left\{K_t(j), N_{S,t}(j), N_{H,t}(j)\right\} \tag{11}$$

The firm, assumed to be a price taker, chooses $N_{S,t}$, $N_{H,t}$ and K_t to maximizes the following expected profit function.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_S^t \left(\frac{\lambda_{S,t}}{\lambda_{S,0}}\right)\left\{Y_t(j) - w_{S,t}N_{St}(j) - w_{H,t}N_{Ht}(j) - r_t K_t(j)\right\} \tag{12}$$

Solving this problem, and imposing symmetry among firms, leads to the following optimality conditions.

$$w_{St} = Z_t F_{NS,t} \tag{13}$$

$$w_{Ht} = Z_t F_{NH,t} \tag{14}$$

$$r_t = Z_t F_{K,t} \tag{15}$$

Equations (13) and (14) are the labor demand conditions for workers from the *saver* and ROT households respectively, whereas equation (15) is the capital demand condition.

2.3 The Government

Each period the government generates revenue through consumption taxes, labor income taxes and capital income taxes. Proceeds from the tax revenue are used for government expenditure G_t. Assuming a balanced budget regime, the government budget constraint is specified as follows.

$$G_t = \tau_t^n \left[(1-\omega) N_{S,t} + \omega N_{H,t} \right] w_t + \tau_t^c \left[(1-\omega) C_{S,t} + \omega C_{H,t} \right] + \tau_t^k (1-\omega)(r_t - \delta) K_t \quad (16)$$

3 The Ramsey Optimal Policy

In this section, I characterize optimal policy by obtaining the second best solution to the optimal tax problem, using the Ramsey approach. The Ramsey planner maximizes the weighted average of utilities of both types of households, subject to the private sector equilibrium conditions and the aggregate resource constraints. The specific welfare function for this economy is given as follows.

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\omega) U_{S,t} + \omega U_{H,t} \right\} \quad (17)$$

Following Lucas and Stokey (1983), I solve the Ramsey problem using the primal approach. In doing so, I derive the Present Value Implementability Constraint (PVIC), which imposes restrictions on the planner by taking into account the private sector equilibrium conditions. This constraint, details of which is presented in Appendix A, reads as follows.

$$(1-\omega) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_S^t \left\{ U_{CS,t} C_{S,t} + U_{NS,t} N_{S,t} \right\} + \omega \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t \left\{ U_{CH,t} C_{H,t} + U_{NH,t} N_{H,t} \right\} = (1-\omega) U_{CS,0} K_0 \left(\frac{1}{1+\tau_0^c} \right) \quad (18)$$

The planner is further constrained by the availability of resources in the economy. In the aggregate, resources of the economy are determined by the sum of the weighted consumption from each household type, net investment from the *saver* households and total government expenditure. The resource constraint is specified as follows.

$$(1-\omega) C_{S,t} + \omega C_{H,t} + (1-\omega) K_{t+1} + G_t = Z_t F(K_t, N_{S,t}, N_{H,t}) + (1-\delta)(1-\omega) K_t \quad (19)$$

The planner therefore chooses $C_{S,t}$, $C_{H,t}$, $N_{S,t}$, $N_{H,t}$, and K_{t+1} to maximize equation (17) subject to conditions (18) and (19). Assuming that $\beta^t = \beta_S^t$ and $\beta_H^t = 1$, the solution to the planner's problem yields the following conditions.

$$\frac{(1-\omega) \left\{ U_{NS,t} + \Omega \left(U_{NNS,t} N_{S,t} + U_{NS,t} \right) \right\}}{U_{CS,t} + \Omega \left(U_{CCS,t} C_{S,t} + U_{CS,t} \right)} = -Z_t F_{NS,t} \quad (20)$$

$$\frac{\omega U_{NH,t} + \omega \Omega \left(U_{NNH,t} N_{H,t} + U_{NH,t} \right)}{u_{cH,t} + \Omega \left(u_{ccH,t} c_{H,t} + u_{cH,t} \right)} = -Z_t F_{NH,t} \quad (21)$$

$$1 = \frac{\beta \mathbb{E}_t \Lambda_{t+1} \left[Z_{t+1} F_{K,t+1} + (1-\omega)(1-\delta) \right]}{(1-\omega) \Lambda_t} \tag{22}$$

where Ω is defined as the Lagrange Multiplier on (18), and Λ_t the Lagrange Multiplier on (19). Also, $U_{XX,t}$ is the second derivative of U_X for any variable X_t.

4 Analytical Results

This section presents the analytical results of the model, regarding the optimal tax structure. I look at the properties of the long run capital income tax rate, the labor income tax rate and consumption tax. In order to obtain the close form solution to the optimal policy problem, I assume the following preferences for the households.

$$U_{i,t}\left(C_{i,t}, N_{i,t}\right) = logC_{i,t} - \chi \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \tag{23}$$

4.1 Optimal Capital Income Taxation in the Long Run

To obtain the long run capital income tax, I use conditions (22) and (9). Under the assumption that $\varphi = 0$, implying linearity in the disutility of labor, we obtain the following relationship at the steady state.

$$r\left(1-\tau^k\right) = \frac{ZF_k}{(1-\omega)} \tag{24}$$

Condition (24) is the market clearing condition in the capital market, based on which I make a proposition regarding the optimal capital income tax.

PROPOSITION 1: In an economy without the ROT consumers, the optimal capital income tax is zero in the long run. In the presence of the ROT consumers however, the optimal capital income tax is not only negative in the long run, it also falls for larger shares of these households.

Proof: From equation (24), if we substitute in condition (15), we obtain

$$\tau^k = \frac{-\omega}{(1-\omega)} \tag{25}$$

Setting $\omega = 0$ results in $\tau^k = 0$. This is a standard outcome in a wide range of work in the literature (see for example, Judd, 1985; Chamley, 1986; Atkeson *et al*, 1999; and Chari et.

al, 1991). With $\omega > 0$, the capital income tax rate is negative. As ω rises the tax rate falls even more. *Q.E.D*

In the standard model without ROT consumers, capital income tax distorts peoples behavior with regards to their savings and consumption decisions. The tax therefore constitutes an inter-temporal wedge between current and future consumption. And, as noted by Judd (1985) and Chamley (1986), this wedge grows with time, especially if we assume a dynamic Ramsey framework that features infinitely lived agents. Beyond this inter-temporal wedge, the tax also distorts the capital market by creating a wedge between the real rental rate of capital (r) and the marginal product of capital (ZF_k). Eliminating this distortion requires that the capital income tax is set equal to zero.

The presence of the ROT consumers introduces further distortions in the capital market. This renders the zero capital tax condition sub-optimal. To elaborate on this, suppose that $\tau^k = 0$. Equation (24) reverts to $r = \frac{ZF_k}{(1-\omega)}$. When $\omega > 0$, the marginal product of capital is lower relative to the real rental rate of capital. For that to happen, the capital stock has to increase. Therefore, to engender a rise in capital formation, the capital income tax is lowered. And, since the capital income tax rate is already zero, a subsidy (negative tax) becomes necessary.

4.2 Optimal Labor Income Taxation

To derive the labor income tax, consider equations (20) and (7). Using the household preferences (equation 23), we obtain the following relationship for the labor income tax, in terms of the share of the ROT consumers and consumption taxes.

$$\tau_t^n = 1 - \frac{1 + \tau_t^c}{(1-\omega)(1+\Omega)} \tag{26}$$

As shown in condition (26), smoothness of the labor income tax depends on the presence of consumption taxes. The size of the labor income tax, on the other hand, depends on both the share of the ROT consumers and the size of the consumption tax. Combining equations (5) (6), (20) and (21) we obtain the relationship for the consumption tax as follows.

$$\tau_t^c = \left(\frac{\omega}{1-\omega}\right)\frac{\Lambda_t}{\lambda_{H,t}} - 1 \tag{27}$$

where $\lambda_{H,t} = \left(\frac{\omega}{1-\omega}\right)\lambda_{S,t}$. From condition (27), we can see that volatility in the consumption tax depends on the behavior of the Lagrange multipliers (Λ_t and $\lambda_{H,t}$), which are time variant. This leads me to the next proposition.

PROPOSITION 2: Suppose consumption is not taxed, the labor income tax rate is constant over the business cycle. In the presence of consumption tax, however, the labor income tax rate is non-constant. And, this is true regardless of whether there are ROT consumers in the economy or not.

Proof: From equation (26), suppose $\tau_t^c = 0$, the labor income tax is obtained as $\tau_t^n = 1 - \frac{1}{(1-\omega)(1+\Omega)}$. The constant labor income tax condition is a standard outcome in a wide range of work in the literature. *Q.E.D*

The smoothing properties of the labor income tax is consistent with the concept of wedge smoothing, which emphasizes smoothing the tax wedge over the business cycle. The tax wedge (tw_t) for this specific economy is obtained as follows.

$$tw_t = \left(\frac{\tau_t^n + \tau_t^c}{1 + \tau_t^c} \right) \tag{28}$$

The wedge constitutes an imperfection in the economy which, ideally, should be stabilized over the business cycle. In a case where $\tau_t^c = 0$, the labor income tax is the only source of the wedge. Smoothing the wedge therefore calls for smoothing the labor income tax. However, if $\tau_t^c > 0$, the constant labor income tax condition is not optimal, given that τ_t^c is non-constant over the business cycle. Since the consumption tax accounts for part of the volatility in the tax wedge, the labor income tax adjusts correspondingly over the business cycle in order to offset the effects of the varying consumption tax on the tax wedge.

PROPOSITION 3: In the presence of the ROT consumers in the economy, the labor income tax falls for larger shares of these households. Consumption tax, on the other hand, rises for larger shares of these households.

Proof: From equations (26) and (27), τ_t^c rises when the share of ROT consumers (ω) rises. At the same time, τ_t^n falls partly due to the direct effects the rise in ω, and partly due to the rise in τ_t^c. *Q.E.D*

Putting together propositions 1 and 3 leads to an important conclusion, which is a central part of this paper. That is, in the long run, income, which includes capital income and labor income, is taxed at a lower rate if the share of ROT consumers in the economy is larger. Consumption on the other hand is taxed at a higher rate for larger shares of these households. In other words, for economies with larger shares of ROT consumers, it is optimal to implement a tax system that puts less weight on income taxes (and more weight on consumption taxes) relative to an economy with a smaller share of these households. The average savings rate in the economy is lower if the fraction of ROT consumers is larger, everything else constant. This leads to a lower capital per worker in the long run. To encourage more savings by the relatively fewer *saver* households, tax policy is designed to put less weight on income taxes as opposed to consumption taxes. Income taxes are generally more inimical to capital formation relative to consumption taxes.

5 Quantitative Results

In this section, I present numerical results of the optimal policy problem. Specifically, I compare allocations and smoothing properties of taxes on consumption, the labor income and capital income across three different specifications of the model. I also present the empirical counterparts for the US economy. The first model, referred to as *Baseline 1,* is the standard model that abstracts from both the consumption tax and ROT consumers, but features the labor income tax and capital income tax. The second model, *Baseline 2*, accounts for the consumption tax but abstracts from ROT consumers. This is also a standard model with all three taxes included. The third model, called the ROT model, accounts for the ROT consumers in the economy and features all three taxes. Before proceeding to analyze the results, I present the model calibration in what follows.

5.1 Calibration

To calibrate the model, I assume the following Cobb-Douglass production technology, together with the household preferences presented in the preceding section (see equation (23)).

$$Y_t = Z_t K_t^{\alpha} N_t^{(1-\alpha)} \tag{29}$$

where $\alpha = (1-\omega)\psi$. N_t is a geometric average of $N_{H,t}$ and $N_{S,t}$ given as follows.

$$N_t = N_{H,t}^{\omega} N_{S,t}^{(1-\omega)} \tag{30}$$

Assuming a quarterly time unit, the discount factor of the planner (β) and that of the *saver* household (β_S) are set at 0.99. Since the ROT consumers do not save I set $\beta_H = 1$. Also, I assume linearity in the disutility of labor. Therefore φ is set equal to 0. The parameter ψ is set in order to obtain α, which is the elasticity of output with respect to capital, equal to 0.34. The deprecation rate of physical capital is set at 0.026, which conforms to an annual rate of about 10%, a value which is within a plausible range estimated to match the investment-capital ratio in the National Income and Product Accounts (NIPA) from 1960 to 2015. The exogenous production technology, Z_t, and government spending, G_t, are governed by the following AR(1) process.

$$logZ_t = \rho_Z log\frac{Z_{t-1}}{\overline{Z}} + \sigma_{z,t} \tag{31}$$

$$logG_t = \rho_Z log\frac{G_{t-1}}{\overline{G}} + \sigma_{g,t} \tag{32}$$

where $\sigma_{z,t}$ and $\sigma_{z,t}$ are the error terms, standard deviation of which are given as v_z and v_g respectively. v_z and v_g are independently set to obtain the standard deviation of output equal to 0.015. This conforms to that of output for the US over the period 1964:1-2007:4. The mean for the production technology, $\overline{Z}$, is set equal to 1, whereas that of government spending, $\overline{G}$, is set equal to 20% of GDP; a value that is empirically plausible for the US economy. The rest of the parameter sets (shown in Table B.1) are chosen to conform to the literature.

5.2 Optimal Taxation in the Long Run

Table 2 presents moment conditions of fiscal policy variables following total factor productivity (TFP) and government spending shocks. Results are presented for the different versions of the model alongside their empirical counterparts. From this table, a number of observations are made regarding the long run optimal tax structure and the short run cyclical properties, which generally conform to the conclusions in the analytical section.

First, in the economy where consumption is not taxed (*Baseline 1*), both the labor income tax and the capital income tax are very stable. Specifically, the labor income tax rate is constant over the business cycle. And, as a derivative outcome, the tax wedge is also constant over the business cycle. With the introduction of consumption tax (*Baseline 2* and *ROT* model), both the labor income tax and the capital in tax become relatively more volatile. The consumption tax adds to the wedge that was created by the labor income tax. This situation renders the constant labor income tax condition sub-optimal. In order to achieve stability in the wedge over the business cycle, the labor income tax has to adjust in order to offset the effects of changes in the consumption tax. Interestingly, the tax wedge that arises from the optimal policy models, although almost identical to the empirical counterpart, is very stable over the business cycle. In other words, stability in the tax wedge is an inherent outcome in the optimal tax design.

Secondly, when we compare results under *Baseline 2* to that of the *ROT* model, both of which feature consumption taxes, it shows the following: The optimal long run capital income tax is zero in the absence of the *ROT* consumers (*Baseline 2*). However, when we account for the *ROT* consumers (*ROT* model), the capital income tax is less than zero (a subsidy).

This is also consistent with the results in the analytical section. Moreover, the optimal labor income tax is lower under the ROT model than under *Baseline 2*. Consumption tax, on the other hand, is higher under the ROT model than under *Baseline 2*. To conclude, in the presence of the ROT consumers, the planner lowers income taxes (a subsidy in the case of capital income) while at the same time raising the consumption tax. This is done in order to induce more savings by the relatively fewer *saver* households, which compensates for the potential loss in total savings, and for that matter investment. The rise in the consumption tax offsets the fall in total tax revenue resulting from the lower income taxes.

Table 2: Moment Conditions of Fiscal Policy Variables Following a Joint TFP and Government Expenditure Shocks.

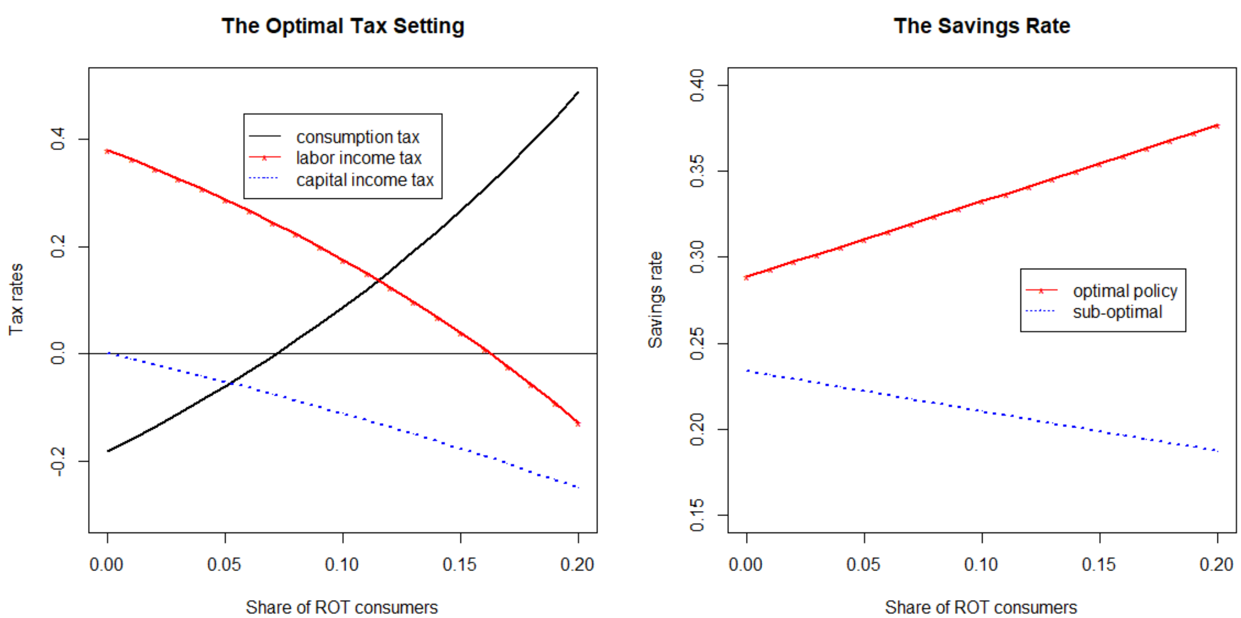
	Model			US Economy
	Baseline 1	Baseline 2	ROT	
Consumption tax				
Mean	–	−18.13	8.68	8.36
Std dev.	–	.20	.23	.96
Corr w/Y	–	.53	.49	−.75
Labor income tax				
Mean	17.78	37.88	17.54	17.78
Std dev.	.00	.15	.17	3.68
Corr w/Y	–	−.53	−.49	.77
Capital income tax				
Mean	.00	.00	−11.11	28.85
Std dev.	.10	1.62	1.58	3.92
Corr w/Y	−.54	.70	.66	−.75
Tax wedge				
Mean	17.78	24.12	24.12	24.14
Std dev.	.00	.00	.00	3.05
Corr w/Y	–	–	–	.69

Note: *Baseline 1* is a standard model that abstracts from both consumption taxes and ROT consumers. *Baseline 2* includes consumption taxes but abstracts from ROT consumers. The ROT model accounts for the ROT consumers in the economy and includes all taxes. For the ROT model I set $\omega = 0.1$, which corresponds to the fraction of households in the US with a net-worth of zero or less (Federal Reserve Board's *Survey of Consumer Finances*, 2017). Estimates for the US economy are constructed from the McDaniel's tax series. This is annual data from 1950-2015. Means and standard deviations are in percentages.

5.3 Dynamic Properties of the Optimal Tax Structure

In this subsection, I present the long run properties of the optimal tax structure in a dynamic framework. Specifically, I look at how the optimal tax structure evolves automatically in response to changes in the share of the ROT consumers in the economy. We can think of this change as countries being characteristically different in terms of the share of the ROT consumers. Or, for a single country, we can think of the fraction of these consumers changing over time. Figure 3 present the optimal tax structure and the resultant savings rate in this dynamic environment. The counterpart savings rate, obtained outside the optimal policy setting, is also presented for comparison (see footnote 2).[2]

Figure 3: Optimal Combination of Taxes and the Corresponding Savings Rate for Varying Shares of the ROT consumers



As shown in this figure, income taxes, which include capital income tax and labor income tax, fall for larger shares of the ROT consumers, whereas consumption tax rises in response to this change. This implies that, for countries with larger shares of the ROT consumers, optimal policy prescription is to tax income at a relatively lower rate (and consumption at a relatively higher rate) compared to countries with smaller shares of these households.

[2]To determine the savings rate outside the optimal setting, suppose savings equals investment in the long run. Given investment per *saver* household $I_t = K_{t+1} - (1-\delta) K_t$, savings per *saver* household s_t at the steady state is obtained as $s = \delta K$, where K is capital holding by each *saver* household. Defining aggregate savings as S_t, the savings rate in the economy is obtained as $\frac{S}{Y} = \frac{(1-\omega)s}{Y}$. For a given amount of savings by each *saver* household, the savings rate in the economy is lower the larger the share of the ROT consumers. If we assume a Cobb-Douglas production technology, we obtain $\frac{S}{Y} = (1-\omega)\,\delta \left(\frac{K}{N}\right)^{(1-\alpha)}$.

Setting taxes relatively lower on labor income and capital income encourages savings and investment, which is particularly critical when the share of *savers* in the economy falls. And, as shown in the second panel, the savings rate under the optimal policy rises in response to this schedule. Outside the optimal policy seting however, the savings rate falls for larger shares of the ROT consumers.

6 Summary and Conclusions

In this paper, I explore the dynamics in the long run optimal tax structure in response to changes in the composition of households in terms of how many save and invest in capital (*savers*) and how many do not, referred to in this paper as *Rule-of-Thumb (ROT)* consumers. Specifically, I account for the ROT consumers in a traditional neoclassical model and use the resultant framework to analyze changes in the optimal tax structure for varying shares of these households. In doing so, I compare the rates and smoothing properties of consumption tax, the labor income tax and the capital income tax across different models reflecting different shares of the ROT consumers. I also analyze trends in these taxes, moving from an economy with a lower fraction of ROT consumers to one with a larger fraction of these households. I find the following.

In the standard model without ROT consumers, the optimal capital income tax rate is zero in the long run, which is consistent with a large number of work in the literature (see Judd, 1985, Chamley, 1986 and Atkeson *et al*, 1999). However, once we account for the ROT consumers, the optimal capital income tax rate is not only negative, it also falls for larger shares of these households. This schedule achieves the level savings and investment that is consistent with optimum welfare, given the relatively fewer *savers* in the economy. Secondly, smoothing properties of the labor income tax is contingent on whether we levy consumption taxes or not. In the standard model without consumption taxes, the optimal labor income tax rate is constant over the business cycle. However, once we introduce consumption taxes, the labor income tax becomes variable over the business cycle, irrespective of whether the ROT consumers are present or not. The labor income tax adjusts over the business cycle to offset the excess volatility in the tax wedge brought about by the introduction of the consumption tax.

In addition, the results favor a tax system that puts less emphasis on income taxes (and more emphasis on consumption taxes) for economies with larger shares of the ROT consumers. The savings rate is lower if the fraction of ROT consumers is larger, everything else constant. To encourage more savings by the relatively fewer *saver* households, tax policy is designed to put less weight on income taxes, which are more inimical to capital formation as opposed consumption taxes.

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A Mathematical Appendix

A.1 The Present Value Implementability Constraint (PVIC)

From the budget constraints of the households we have the following.

$$(1+\tau_t^c)\,C_{S,t}+K_{t+1}=(1-\tau_t^n)\,w_tN_{S,t}+\left\{(1-\delta)+r_t\left(1-\tau_t^k\right)\right\}K_t \tag{A.1}$$

$$(1+\tau_t^c)\,C_{H,t}=(1-\tau_t^n)\,w_tN_{H,t} \tag{A.2}$$

Multiplying by the weight of each household category, we obtained the following in the aggregate.

$$(1-\omega)\left[(1-\tau_t^n)\,w_tN_{S,t}+\left\{(1-\delta)+r_t\left(1-\tau_t^k\right)\right\}K_t\right]+\omega\,(1-\tau_t^n)\,w_tN_{H,t}= \tag{A.3}$$

$$(1-\omega)\left[\left(1+\tau_t^c\right)C_{S,t}+K_{t+1}\right]+\omega\left(1+\tau_t^c\right)C_{H,t}$$

Introducing $\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}$ and $\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{CH,t}$ into (A.3), we obtain the following

$$\begin{aligned}
&\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\left(1-\tau_t^n\right)w_t N_{S,t}+\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{cH,t}\omega\left(1-\tau_t^n\right)w_t N_{H,t}\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\left\{\left(1-\delta\right)+r_t\left(1-\tau_t^k\right)\right\}K_t-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\left(1+\tau_t^c\right)C_{S,t}\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{CH,t}\omega\left(1+\tau_t^c\right)C_{H,t}-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)K_{t+1}=0
\end{aligned}\tag{A.4}$$

Substituting in equation (7), (8) and (9) we obtain the following.

$$\begin{aligned}
&\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\left(-\frac{U_{NS,t}}{U_{CS,t}}\right)\left(1+\tau_t^c\right)N_{S,t}+\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{CH,t}\omega\left(-\frac{U_{NH,t}}{U_{CH,t}}\right)\left(1+\tau_t^c\right)N_{H,t}\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\frac{1}{\beta_{\mathbf{S}}}\left(\frac{U_{CS,t-1}}{U_{CS,t}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t-1}^c}\right)K_t-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)\left(1+\tau_t^c\right)C_{S,t}\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{CH,t}\omega\left(1+\tau_t^c\right)C_{H,t}-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(1-\omega\right)K_{t+1}=0
\end{aligned}\tag{A.5}$$

Dividing equation (A.5) by $\left(1+\tau_t^c\right)$ and simplifying, we obtain

$$\begin{aligned}
&-\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{NS,t}N_{S,t}-\omega\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{NH,t}N_{H,t}-\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}C_{S,t}\\
&-\omega\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t U_{CH,t}C_{H,t}+\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^{t-1}U_{CS,t-1}\left(\frac{1}{1+\tau_{t-1}^c}\right)K_t\\
&-\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(\frac{1}{1+\tau_t^c}\right)K_{t+1}=0
\end{aligned}\tag{A.6}$$

If we rearrange (A.6) we obtain the following.

$$\begin{aligned}
&\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t\left[U_{CS,t}C_{S,t}+U_{NS,t}N_{S,t}\right]+\omega\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t\left[U_{CH,t}C_{H,t}+U_{NH,t}N_{H,t}\right]\\
&=\left(1-\omega\right)\left\{U_{CS,0}\left(\frac{1}{1+\tau_0^c}\right)K_0+\sum_{t=1}^{\infty}\beta_S^{t-1}U_{CS,t-1}\left(\frac{1}{1+\tau_{t-1}^c}\right)K_t-\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t U_{CS,t}\left(\frac{1}{1+\tau_t^c}\right)K_{t+1}\right\}
\end{aligned}\tag{A.7}$$

From this condition, we obtain the Present Value Implementability Constraint (PVIC) as follows.

$$\left(1-\omega\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta_S^t\left[U_{CS,t}C_{S,t}+U_{NS,t}N_{S,t}\right]+\omega\mathbb{E}_0\sum_{t=0}^{\infty}\beta_H^t\left[U_{CH,t}C_{H,t}+U_{NH,t}N_{H,t}\right]=\left(1-\omega\right)U_{CS,0}\left(\frac{1}{1+\tau_0^c}\right)K_0\tag{A.8}$$

A.2 The Planners First Order Conditions

$$(1-\omega)\,U_{CS,t} + (1-\omega)\,\Omega\,(U_{CCS,t}C_{S,t} + U_{CS,t}) - (1-\omega)\,\Lambda_t = 0 \tag{A.9}$$

$$\omega U_{CH,t} + \omega\Omega\,(U_{CCH,t}C_{H,t} + U_{CH,t}) - \omega\Lambda_t = 0 \tag{A.10}$$

$$(1-\omega)\,U_{NS,t} + (1-\omega)\,\Omega\,(U_{NNS,t}N_{S,t} + U_{NS,t}) + \Lambda_t z_t F_{NS,t} = 0 \tag{A.11}$$

$$\omega U_{NH,t} + \omega\Omega\,(U_{NNH,t}C_{H,t} + U_{NH,t}) + \Lambda_t Z_t F_{NH,t} = 0 \tag{A.12}$$

$$\beta\Lambda_{t+1}Z_{t+1}F_{k,t+1} + \beta\,(1-\delta)\,(1-\omega)\,\Lambda_{t+1} - (1-\omega)\,\Lambda_t = 0 \tag{A.13}$$

B Tables and Figures

Table B.1: Parameters sets

Parameter	Model		
	Baseline 1	Baseline 2	ROT
β	0.99	0.99	0.99
β_S	0.99	0.99	0.99
β_H	–	–	1.00
σ	2.00	2.00	2.00
ω	0	0	0.1
α	0.34	0.34	0.34
ψ	α	α	$\frac{\alpha}{(1-\omega)}$
φ	0	0	0
δ	0.026	0.026	0.026
ρ_g	0.80	0.80	0.80
ρ_Z	0.95	0.95	0.95
υ_g	0.0067	0.062	0.062
υ_Z	0.0023	0.0017	0.0017

Table B.2: Variance decomposition for output and fiscal policy variables.

	Model		
	Baseline 1	Baseline 1	NC
Output			
Govenment expenditure	49.79	49.98	45.44
TFP shock	50.21	50.02	54.56
consumption tax			
Govenment expenditure	–	94.89	94.92
TFP shock	–	5.11	5.08
Labor income tax			
Govenment expenditure	13.38	94.89	94.92
TFP shock	86.62	5.11	5.08
Capital income tax			
Govenment expenditure	95.76	99.97	99.97
TFP shock	4.24	.03	.03

Figure B.1: Impulse responses to a TPF shock

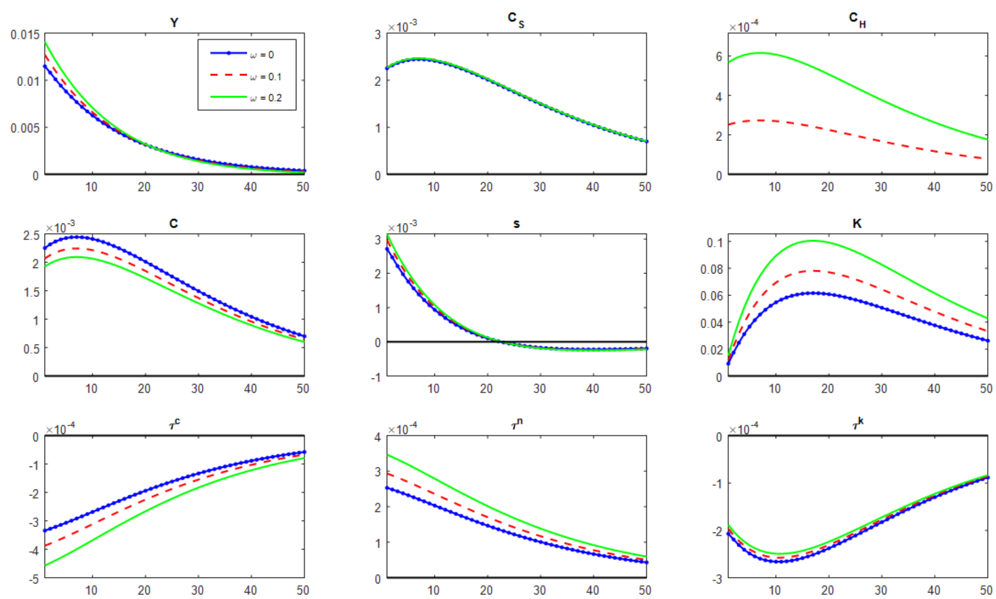

Figure B.2: Impulse responses to a government expenditure shock